Class XI Session 2025-26
Subject - Mathematics
Sample Question Paper - 7

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question paper contains 38 questions. All questions are compulsory.
2. This Question paper is divided into five Sections - A, B, C, D and E.
3. In Section A, Questions no. 1 to 18 are multiple choice questions (MCQs) and Questions no. 19 and 20 are Assertion-Reason based questions of 1 mark each.
4. In Section B, Questions no. 21 to 25 are Very Short Answer (VSA)-type questions, carrying 2 marks each.
5. In Section C, Questions no. 26 to 31 are Short Answer (SA)-type questions, carrying 3 marks each.
6. In Section D, Questions no. 32 to 35 are Long Answer (LA)-type questions, carrying 5 marks each.
7. In Section E, Questions no. 36 to 38 are Case study-based questions, carrying 4 marks each.
8. There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and one subpart each in 2 questions of Section E.
9. Use of calculators is not allowed.

Section A

1. If tan θ = 3 and θ lies in third quadrant, then the value of sin θ is **[1]**

a) $\frac{-3}{\sqrt{10}}$ b) $-\frac{1}{\sqrt{10}}$

c) $\frac{1}{\sqrt{10}}$ d) $\frac{3}{\sqrt{10}}$

2. Let f : R → R be defined by $f(x) = \begin{cases} 2x : x > 3 \\ x^2 : 1 < x \leq 3 \\ 3x : x \leq 1 \end{cases}$ **[1]**

Then f (–1) + f (2) + f (4) is

a) 6 b) 9

c) 5 d) 14

3. A bag contains 5 brown and 4 white socks. A man pulls out two socks. The probability that these are of the same colour is **[1]**

a) $\frac{30}{108}$ b) $\frac{5}{108}$

c) $\frac{48}{108}$ d) $\frac{18}{108}$

4. If f(x) = $x^{100} + x^{99}$ ... + x + 1, then f'(1) is equal to: **[1]**

a) 5050 b) 5049

c) 50051 d) 5051

5. The coordinates of the foot of the perpendicular drawn from the point P(3, 4, 5) on the yz-plane are [1]

a) (3, 0, 5)
b) (3, 0, 0)
c) (0, 4, 5)
d) (3, 4, 0)

6. $\{C_1 + 2C_2 + 3C_3 + ... + nC_n\} = ?$ [1]

a) $(n + 1) \cdot 2^n$
b) $n \cdot 2^n$
c) $(n - 1) \cdot 2^n$
d) $n \cdot 2^{n-1}$

7. If $z = \frac{1}{(1-i)(2+3i)}$, then |z| = [1]

a) $1/\sqrt{26}$
b) $4/\sqrt{26}$
c) 1
d) $5/\sqrt{26}$

8. The domain of function f : R → R defined by $f(x) = \sqrt{x^2 - 3x + 2}$ is [1]

a) $(-\infty, 1] \cup [2, \infty)$
b) $(-\infty, 1]$
c) [1, 2]
d) $[2, \infty]$

9. Solve the system of inequalities: $-15 < \frac{3(x-2)}{5} \leq 0$ [1]

a) -23 < x < 23
b) -23 < x $\leq$ 2
c) -13 < x < 13
d) -13 < x < 2

10. $\sin(40^\circ + \theta)\cos(10^\circ + \theta) - \cos(40^\circ + \theta)\sin(10^\circ + \theta)$ is equal to [1]

a) 2
b) $\frac{\sqrt{3}}{2}$
c) $\frac{1}{3}$
d) $\frac{1}{2}$

11. Which of the following is a set? [1]

A. A collection of vowels in English alphabets is a set.
B. The collection of most talented writers of India is a set.
C. The collection of most difficult topics in Mathematics is a set.
D. The collection of good cricket players of India is a set.

a) C
b) B
c) A
d) D

12. Sum of an infinite G.P. is $\frac{5}{4}$ times the sum of all the odd terms. The common ratio of the G.P. is [1]

a) $\frac{2}{3}$
b) $\frac{1}{4}$
c) 4
d) $\frac{1}{3}$

13. The number of terms in the expansion of $(\sqrt{x} + \sqrt{y})^8 + (\sqrt{x} - \sqrt{y})^8$ is [1]

a) 8
b) 9
c) 5
d) 7

14. The solution set for (x + 3) + 4 > -2x + 5: [1]

a) $(2, \infty)$
b) $(-\infty, 2)$
c) $(-\infty, -2)$
d) $\left(\frac{-2}{3}, \infty\right)$

15. The set of all prime numbers is [1]

a) an infinite set
b) a singleton set
c) a multi set
d) a finite set

16. If $\sin\theta = \frac{-4}{5}$, and θ lies in third quadrant then the value of $\cos\frac{\theta}{2}$ is [1]

a) $-\frac{1}{\sqrt{5}}$
b) $\frac{1}{\sqrt{10}}$
c) $\frac{1}{5}$
d) $-\frac{1}{\sqrt{10}}$

17. The value of $(-1+\sqrt{-3})^2 + (-1-\sqrt{-3})^2$ is [1]

a) 4
b) -2
c) -4
d) 8

18. The number of ways in which 6 men can be arranged in a row so that three particular men are consecutive, is [1]

a) $3! \times 3!$
b) $4!$
c) $4! \times 3!$
d) $2! \times 3!$

19. **Assertion (A):** The collection of all natural numbers less than 100' is a set. [1]

Reason (R): A set is a well-defined collection of the distinct objects.

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

20. **Assertion (A):** The mean deviation about the mean for the data 4, 7, 8, 9, 10, 12, 13, 17 is 3. [1]

Reason (R): The mean deviation about the mean for the data 38, 70, 48, 40, 42, 55, 63, 46, 54, 44 is 8.5.

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

Section B

21. Let A = {a, b, c, d}, B = {c, d, e} and C = {d, e, f, g}. Then verify the identity: $(A \times B) \cap (B \times A) = (A \cap B) \times (A \cap B)$ [2]

OR

If R is the relation "less than" from A = {1, 2, 3, 4, 5} to B = {1, 4, 5}. Write down the set of ordered pairs corresponding to R. Find the inverse of R.

22. Evaluate : $\lim_{x \to 1}\left(\frac{1}{x-1} - \frac{2}{x^2-1}\right)$. [2]

23. Find the centre and radius of the circle. $x^2 + y^2 + 6x - 4y + 4 = 0$ [2]

OR

Find the equation of a circle with centre (-1, 2) and radius 4.

24. Write the set in the roster form: $F = \{x \mid x^4 - 5x^2 + 6 = 0, x \in R\}$ [2]

25. Find the equation of the straight line passing through (-2, 3) and inclined at an angle of 45° with the x-axis. [2]

Section C

26. Draw the graph of the function |x - 2| [3]

27. Solve 3x + 8 > 2 when [3]

(i) x is integer

(ii) x is a real number

28. How many four digit numbers divisible by 4 can be made with the digits 1, 2, 3, 4, 5 if the repetition of digits is not allowed? **[3]**

OR

If A and B be the points (3, 4, 5) and (-1, 3, -7), respectively, find the equation of the set of points P such that $PA^2 + PB^2 = k^2$, where k is a constant.

29. Find the coefficient of x^6 in the expansion of $\left(3x^2 - \frac{1}{3x}\right)^9$. **[3]**

OR

Using binomial theorem, expand: $\left(x^2 - \frac{2}{x}\right)^7$.

30. Find the multiplicative inverse of the complex number $= \sqrt{5} + 3i$ **[3]**

OR

Find the real numbers x and y if (x - iy) (3 + 5i) is the conjugate of -6 - 24i.

31. For any two sets A and B, prove that $A \cup B = A \cap B \Leftrightarrow A = B$. **[3]**

Section D

32. A bag contains 6 red, 4 white and 8 blue balls. If three balls are drawn at random, find the probability that: **[5]**

  i. one is red and two are white
  ii. two are blue and one is red
  iii. one is red.

33. Evaluate: $\lim_{x \to 0} \frac{1 - \cos x \cos 2x \cos 3x}{\sin^2 2x}$ **[5]**

OR

Differentiate $\frac{\cos x}{x}$ from first principle.

34. Find four numbers in GP, whose sum is 85 and product is 4096. **[5]**

35. Prove that: $\sin 5x = 5 \sin x - 20 \sin^3 x + 16 \sin^5 x$. **[5]**

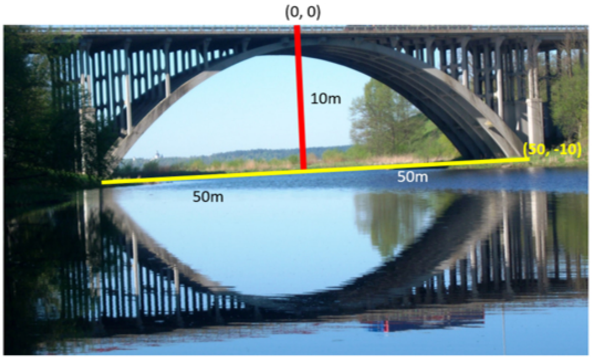
OR

Prove that: $\cos 20° \cos 40° \cos 60° \cos 80° = \frac{1}{16}$.

Section E

36. **Read the following text carefully and answer the questions that follow:** **[4]**

The girder of a railway bridge is a parabola with its vertex at the highest point, 10 m above the ends. Its span is 100 m.



  i. Find the coordinates of the focus of the parabola. (1)
  ii. Find the equation of girder of bridge and find the length of latus rectum of girder of bridge. (1)
  iii. Find the height of the bridge at 20m from the mid-point. (2)

OR

Find the radius of circle with centre at focus of the parabola and passes through the vertex of parabola. (2)

37. **Read the following text carefully and answer the questions that follow:** **[4]**

An analysis of monthly wages paid to workers in two firms A and B, belonging to the same industry, gives the following results:

Particulars	Firm A	Firm B
No. of wage earners	586	648
Mean of monthly wages	₹ 5253	₹ 5253
Variance of the distribution of wages	100	121

i. Which firm A or B shows greater variability in individual wages? (1)

ii. Find the standard deviation of the distribution of wages for frim B. (1)

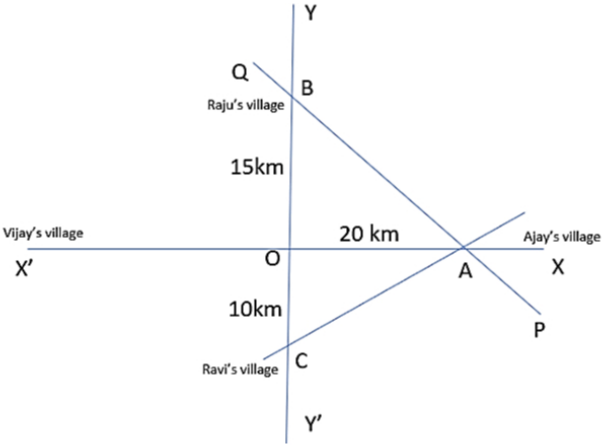
iii. Find the coefficient of variation of the distribution of wages for firm A. (2)

OR

Find the amount paid by firm A. (2)

38. **Read the following text carefully and answer the questions that follow:** **[4]**

Villages of Vijay and Ajay are 70km apart and are situated on Delhi Agra highway as shown in the following picture. Another highway YY' crosses Agra Delhi highway at O (0,0). A small local road PQ crosses both highways at Points A and B such that OA =20km and OB=15 km. Also, the villages of Raju and Ravi are on the smaller highway YY'. Raju's village is 15km from O and that of Ravi's is 10km from O



i. Find the equation of line AB? (1)

ii. Find the equation of line AC? (1)

iii. Find the area of triangle ABC. (2)

OR

iv. What is the length of line AC? (2)

Solution

Section A

1. **(a)** $\frac{-3}{\sqrt{10}}$

Explanation:

Given that, tan θ = 3 and θ lies in third quadrant

$\Rightarrow \cot\theta = \frac{1}{3}$

We know that,

cosec^2 θ = 1 + cot^2 θ

= $1 + \left(\frac{1}{3}\right)^2 = 1 + \frac{1}{9} = \frac{10}{9}$

$\Rightarrow \sin^2\theta = \frac{9}{10}$

$\Rightarrow \sin\theta = \pm\frac{3}{\sqrt{10}}$

$\Rightarrow \sin\theta = -\frac{3}{\sqrt{10}}$, since θ lies in third quadrant.

2.

(b) 9

Explanation:

Given that,

f(x) = $\begin{cases} 2x : x > 3 \\ x^2 : 1 < x \leq 3 \\ 3x^2 : x \leq 1 \end{cases}$

Now,

f(-1) = 3(-1) = -3 [since -1<1 and f(x) = 3x for x $\leq$ 1]

f(2) = 2^2 = 4 [since 2 < 3 and f(x) = x^2 for 1< x $\leq$ 3]

f(4) = 2(4) = 8 [since 4 > 3 and f(x) = 2x for x > 3]

$\therefore$ f (– 1) + f (2) + f (4) = -3 + 4 + 8 = 9

3.

(c) $\frac{48}{108}$

Explanation:

P (same coloured socks) = P (both brown) + P(both white)

= $\frac{5}{9} \times \frac{4}{8} + \frac{4}{9} \times \frac{3}{8}$

= $\frac{20}{72} + \frac{12}{72}$

= $\frac{32}{72}$

= $\frac{4}{9} = \frac{48}{108}$

4. **(a)** 5050

Explanation:

Given, f(x) = x^{100} + x^{99} ... + x + 1

$\therefore$ f'(x) = $100x^{99}$ + $99x^{98}$ ... + x + 1

So, f'(1) = 100 + 99 + 98 + ... + 1

= $\frac{100}{2}$[2 $\times$ 100 + (100 - 1)(-1)]

= 50[200 - 99] = 50 $\times$ 101

= 5050

5.

(c) (0, 4, 5)

Explanation:

On yz - plane x coordinate of any point is equal to 0.
The coordinates of the foot of the perpendicular drawn from the point P(3, 4, 5) on the yz-plane are (0, 4, 5).

6.

(d) n· 2^{n-1}

Explanation:

$C_1 + 2C_2 + 3C_3 + ... + nC_n = n + 2 \cdot \frac{n(n-1)}{2} + 3 \cdot \frac{n(n-1)(n-2)}{3!} + ... + n$

$= n . [1 + (n - 1) \frac{(n-1)(n-2)}{2!} + ... + 1]$

$= n.[^{(n - 1)}C_0 + {}^{(n - 1)}C_1 + {}^{(n-1)}C_2 + ... + {}^{(n-1)}C_{n-1}]$

$= n .(1 + 1)^{n-1} = n.2^{n-1}$

7. **(a)** $1/\sqrt{26}$

Explanation:

$1/\sqrt{26}$

Let z $= \frac{1}{(1-i)(2+3i)}$

$\Rightarrow z = \frac{1}{2+i-3i^2}$

$\Rightarrow z = \frac{1}{2+i+3}$

$\Rightarrow z = \frac{1}{5+i} \times \frac{5-i}{5-i}$

$\Rightarrow z = \frac{5-i}{25-i^2}$

$\Rightarrow z = \frac{5-i}{25+1}$

$\Rightarrow z = \frac{5-i}{26}$

$\Rightarrow z = \frac{5}{26} - \frac{i}{26}$

$\Rightarrow |z| = \sqrt{\frac{25}{676} + \frac{1}{676}}$

$\Rightarrow z = \frac{1}{\sqrt{26}}$

8. **(a)** $(-\infty, 1] \cup [2, \infty)$

Explanation:

$\because$ f : R $\rightarrow$ R defined by

$f(x) = \sqrt{x^2 - 3x + 2}$

Here, $x^2 - 3x + 2 \geq 0$

$(x - 1)(x - 2) \geq 0$

$x \leq 1$ or $x \geq 2$

$\therefore$ Domain of $f = (-\infty, 1] \cup [2, \infty)$

9.

(b) $-23 < x \leq 2$

Explanation:

$-15 < \frac{3(x-2)}{5} \leq 0$

$\Rightarrow -15 \cdot \frac{5}{3} < \frac{3(x-2)}{5} \cdot \frac{5}{3} \leq 0 \cdot \frac{5}{3}$

$\Rightarrow -25 < (x - 2) \leq 0 + 2$

$\Rightarrow -25 + 2 < x - 2 + 2 \leq 2$

$\Rightarrow -23 < x \leq 2$

10.

(d) $\frac{1}{2}$

Explanation:

$\sin(40^\circ + \theta)\cos(10^\circ + \theta) - \cos(40^\circ + \theta)\sin(10^\circ + \theta)$

sin (A - B)= sin A cos B - cos A sin B, where $(40^\circ + \theta) = A$ and $(10^\circ + \theta) = B$

$= \sin\{(40^\circ + \boldsymbol{\theta}) - (10^\circ + \theta)\} = \sin 30^\circ = \frac{1}{2}$

11.

(c) A

Explanation:

The set is {a, e, i, o, u}

12.

(b) $\frac{1}{4}$

Explanation:

Consider the infinite G.P a, ar, ar^2, ar^3,.... is a G.P with first term a and common ratio r

Then the odd terms a, ar^2, ar^4,.... is again a G.P with first term a and common ratio r^2

We have $S_\infty = \frac{a}{1-r}$

Given $S_\infty = \frac{5}{4}$. Sum of odd terms

$\Rightarrow$ a + ar + ar^2 + ar^3 + ... = $\frac{5}{4}$[a + ar^2 + ar^4 + ...]

$\Rightarrow \frac{a}{1-r} = \frac{5}{4} \cdot \frac{a}{1-r^2}$

$\Rightarrow \frac{1}{1-r} = \frac{5}{4} \cdot \frac{a}{(1-r)(1+r)}$

$\Rightarrow$ 4(1 + r) = 5

$\Rightarrow$ 4r = 1

$\Rightarrow$ r = $\frac{1}{4}$

13.

(c) 5

Explanation:

Therefore,the expansion of $\left\{(\sqrt{x}+\sqrt{y})^8+(\sqrt{x}-\sqrt{y})^8\right\}$

For given bionomial , we use n/2+1 for number of terms ,

where n=8, hence , 4+1=5 terms.

14.

(d) $\left(\frac{-2}{3}, \infty\right)$

Explanation:

(x + 3) + 4 > -2x + 5

$\Rightarrow$ x + 7 > -2x + 5

$\Rightarrow$ x + 7 + 2x > -2x + 5 + 2x

$\Rightarrow$ 3x + 7 > 5

$\Rightarrow$ 3x + 7 - 7 > 5 - 7

$\Rightarrow$ 3x > -2

$\Rightarrow$ x > $\frac{-2}{3}$

$\Rightarrow x \in \left(\frac{-2}{3}, \infty\right)$

15. **(a)** an infinite set

Explanation:

Set A = {2, 3, 5, 7,...} so it is infinite.

16. **(a)** $-\frac{1}{\sqrt{5}}$

Explanation:

Given that $\sin\theta = \frac{-4}{5}$ and θ lies in third quadrant.

$\cos\theta = \sqrt{1-\sin^2\theta}$

= $\sqrt{1-\left(\frac{-4}{5}\right)^2}$

= $\sqrt{1-\frac{16}{25}}$

$= \sqrt{\frac{9}{25}}$
$= \pm\frac{3}{5}$
$\Rightarrow \cos\theta = -\frac{3}{5}$ since θ lies in third quadrant
$\cos\theta = 2\cos^2\frac{\theta}{2} - 1$
$\Rightarrow 2\cos^2\frac{\theta}{2} = 1 - \frac{3}{5} = \frac{2}{5}$
$\Rightarrow \cos^2\frac{\theta}{2} = \frac{1}{5}$
$\Rightarrow \cos\frac{\theta}{2} = \pm\frac{1}{\sqrt{5}}$
$\Rightarrow \cos\frac{\theta}{2} = -\frac{1}{\sqrt{5}}$ (since $\frac{\theta}{2}$ lies in second quadrant)

17.

(c) -4

Explanation:

We have $\sqrt{-3} = \sqrt{-1.3} = \sqrt{-1}\sqrt{3} = \sqrt{3}i$
$\therefore -1 + \sqrt{-3} = -1 + i\sqrt{3} = 2\omega$ and $-1 - \sqrt{-3} = -1 - i\sqrt{3} = 2\omega^2$
$\left[\because \omega = \frac{-1+i\sqrt{3}}{2} \text{ and } \omega^2 = \frac{-1-i\sqrt{3}}{2}\right]$
$(-1+\sqrt{-3})^2 + (-1-\sqrt{-3})^2 = (2\omega)^2 + \left(2\omega^2\right)^2 = 4\omega^2 + 4\omega^4 = 4\left(\omega^2 + \omega^3 \cdot \omega\right)$
$=4\left(\omega^2 + \omega\right) = 4 \times -1 = -4 \; \left[\because \omega^3 = 1, 1 + \omega + \omega^2 = 0\right]$

18.

(c) 4! × 3!

Explanation:

As, it is required that three particular men are consecutive, so, let us consider the three particular men as a single object.
Thus we have, 4 objects, which can be arranged in 4! ways, and the 3 particular men who are consecutive can be arranged in 3! ways among themselves.
So, the number of arrangements in which 6 men can be arranged in a row so that three particular men are consecutive, is
= 4! × 3! = 144

19. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation:

Assertion The collection of all natural numbers less than 100', is a well-defined collection. So, it is a set.

20.

(c) A is true but R is false.

Explanation:

Assertion Mean of the given series
$\bar{x} = \frac{\text{Sum of terms}}{\text{Number of terms}} = \frac{\sum x_i}{n}$
$= \frac{4+7+8+9+10+12+13+17}{8}$ = 10

xi	**|xi - $\bar{x}$|**
4	|4 - 10| = 6
7	|7 - 10| = 3
8	|8 - 10| = 2
9	|9 - 10| = 1
10	|10 - 10| = 0
12	|12 - 10| = 2
13	|13 - 10| = 3
17	|17 - 10| = 7
$\sum x_i$ = 80	$\sum \lvert x_i - \bar{x}\rvert$ = 24

$\therefore$ Mean deviation about mean

$= \frac{\Sigma|x_i-\bar{x}|}{n} = \frac{24}{8}$ = 3

Reason Mean of the given series

$\bar{x} = \frac{\text{Sum of terms}}{\text{Number of terms}} = \frac{\sum x_i}{n}$

$= \frac{38+70+48+40+42+55}{+63+46+54+44}$ = 50

$\therefore$ Mean deviation about mean

$= \frac{\Sigma|x_i-\bar{x}|}{n}$

$= \frac{84}{10}$ = 8.4

Hence, Assertion is true and Reason is false.

Section B

21. Here we have, A = {a, b, c, d}, B = {c, d, e} and C = {d, e, f, g}

To verify: $(A \times B) \cap (B \times A) = (A \cap B) \times (A \cap B)$

From LHS, we have

$(A \times B)$ = {(a, c), (a, d), (a, e), (b, c), (b, d), (b, e), (c, c), (c, d), (c, e), (d, c), (d, d), (d, e)}

$(B \times A)$ = {(c, a), (c, b), (c, c), (c, d), (d, a), (d, b), (d, c), (d, d), (e, a), (e, b), (e, c), (e, d)}

Now, $(A \times B) \cap (B \times A)$ = {(c, c), (c, d), (d, c), (d, d)}

LHS = RHS

So, $(A \times B) \cap (B \times A)$= $(A \cap B) \times (A \cap B)$

Hence Verified.

OR

Here A = {1, 2, 3, 4, 5} and B = {1, 4, 5} a $\in$ A, b $\in$ B

$\therefore$ a < b = 1 < 4, 1 < 5, 2 < 4, 2< 5, 3 < 4, 3 < 5, 4 < 5

$\therefore$ R = {(1, 4), (1, 5), (2, 4), (2, 5), (3, 4), (3, 5), (4, 5)}

Now R^{-1}= {(4, 1), (5,1), (4, 2), (5, 2), (4, 3), (5, 3), (5, 4)}

22. We have:$\lim_{x \to 1}\left(\frac{1}{x-1} - \frac{2}{x^2-1}\right)$

= $\lim_{x \to 1}\left(\frac{1}{x-1} - \frac{2}{(x-1)(x+1)}\right)$

= $\lim_{x \to 1}\left(\frac{x+1-2}{(x-1)(x+1)}\right)$

= $\lim_{x \to 1}\left(\frac{x-1}{(x-1)(x+1)}\right)$

= $\lim_{x \to 1}\left(\frac{1}{x+1}\right)$

= $\frac{1}{1+1} = \frac{1}{2}$

23. Given equation of circle is $x^2 + y^2 + 6x - 4y + 4 = 0$

$\Rightarrow (x^2 + 6x) + (y^2 - 4y) = - 4$

$\Rightarrow (x^2 + 6x + 9 - 9) + (y^2 - 4y + 4 - 4) = - 4$

$\Rightarrow (x^2 + 6x + 9) + (y^2 - 4y + 4) = - 4 + 4 + 9$

$\Rightarrow (x + 3)^2 + (y - 2)^2 = 9$

$\Rightarrow \{x - (- 3)\}^2 + \{y - 2\}^2 = 3^2$

On comparing with $(x - h)^2 + (y - k)^2 = r^2$, we get

h = - 3, k = 2 and r = 3

Hence, centre of circle = (- 3, 2) and radius = 3.

OR

The equation of a circle with centre (h, K) and radius 'r' is given by

$(x - h)^2 + (y - k)^2 = r^2$ ...(i)

Here, (h, k) = (-1, 2) and r =4

$\therefore$ (i) $\Rightarrow \{x - (-1)\}^2 + (y - 2)^2 = 4^2$

$\Rightarrow (x + 1)^2 + (y - 2)^2 = 4^2$

$\Rightarrow x^2 + 2x + 1 + y^2 - 4y + 4 = 16$

$\Rightarrow x^2 + y^2 + 2x - 4y - 11 = 0$

24. Here, $F = \{x | x^4 - 5x^2 + 6 = 0, x \in R\}$

$\therefore x^4 - 5x^2 + 6 = 0 \Rightarrow x^4 - 3x^2 - 2x^2 + 6 = 0$

$\Rightarrow x^2(x^2 - 3) - 2(x^2 - 3) = 0 \Rightarrow (x^2 - 2)(x^2 - 3) = 0$

$\Rightarrow x^2 - 2 = 0$ and $x^2 - 3 = 0$

$\Rightarrow x = \pm\sqrt{2}$ and $x = \pm\sqrt{3}$

$\therefore F = \{-\sqrt{3}, -\sqrt{2}, \sqrt{2}, \sqrt{3}\}$

25. Given that, $m = \tan 45^o = 1$

$x_1 = -2$ and $y_1 = 3$

Substituting these values in $y - y_1 = m(x - x_1)$, we get,

$y - 3 = 1(x + 2)$

$\Rightarrow y - 3 = x + 2$

$\Rightarrow x - y + 5 = 0$

Therefore, the equation of the required line is $x - y + 5 = 0$

Section C

26. Clearly,

$y = |x - 2| = \begin{cases} x - 2, & x - 2 \geq 0 \\ -(x - 2), & x - 2 < 0 \end{cases}$

$= \begin{cases} x - 2, & x \geq 2 \\ 2 - x, & x < 2 \end{cases}$

We know that, a linear equation in x and y represents a line for drawing a line, we need only two points for $y = x - 2$

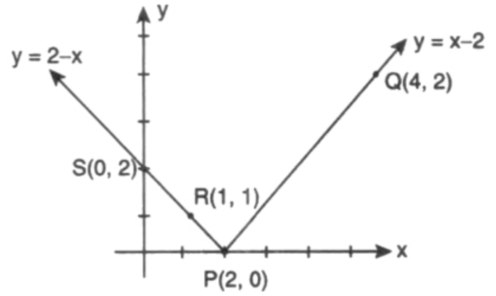
x	2	4
y	0	2

So, plot the points P(2, 0), Q(4, 2) and join PQ to get the graph of $y = x - 2$

for $y = 2 - x$:

x	1	0
y	1	2

Plot the points R(1, 1), S(0, 2) and join RS to get the graph of $y = 2 - x$



27. Here $3x + 8 > 2$

$\Rightarrow 3x > 2 - 8 \Rightarrow 3x > -6$

Dividing both sides by 3, we have

$x > -2$

(i) When x is an integer then values of x that make the statement true are -1, 0, 1, 2, 3, . . . The solution set of inequality is {-1, 0, 1, 2, 3, . . . }

(ii) When x is a real number. The solution set of inequality is $x \in (-2, \infty)$

28. Recall that a number is divisible by 4 if the number formed by the last two digits is divisible by 4. The digits at unit's and ten's places can be arranged as follows:

Th	H	T	O
x	x	1	2
x	x	2	4

x	x	3	2
x	x	5	2

Now, corresponding each such way the remaining three digits at thousand's and hundred's places can be arranged in ${}^3P_2$ ways.

Hence, the required number of numbers = ${}^3P_2 \times 4 = 3! \times 4 = 24$

OR

The equation of the set of points P such that $PA^2 + PB^2 = k^2$, where k is a constant

Given: The points A (3, 4, 5) and B (-1, 3, -7)

$\Rightarrow x_1 = 3, y_1 = 4, z_1 = 5; x_2 = -1, y_2 = 3, z_2 = -7;$

$PA^2 + PB^2 = k^2$ ...(i)

Let the point be P (x, y, z).

Now, by Distance Formula, we know that the distance between two points P (x_1, y_1, z_1) and Q (x_2, y_2, z_2) is given by

$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$.

So, $PA = \sqrt{(3-x)^2 + (4-y)^2 + (5-z)^2}$

And $PB = \sqrt{(-1-x)^2 + (3-y)^2 + (-7-z)^2}$

Now, substituting these values in (i), we have

$[(3 - x)^2 + (4 - y)^2 + (5 - z)^2] + [(-1 - x)^2 + (3 - y)^2 + (-7 - z)^2] = k^2$

$\Rightarrow [(9 + x^2 - 6x) + (16 + y^2 - 8y) + (25 + z^2 - 10z)] + [(1 + x^2 + 2x) + (9 + y^2 - 6y) + (49 + z^2 + 14z)] = k^2$

$\Rightarrow 9 + x^2 - 6x + 16 + y^2 - 8y + 25 + z^2 - 10z + 1 + x^2 + 2x + 9 + y^2 - 6y + 49 + z^2 + 14z = k^2$

$\Rightarrow 2x^2 + 2y^2 + 2z^2 - 4x - 14y + 4z + 109 = k^2$

$\Rightarrow 2x^2 + 2y^2 + 2z^2 - 4x - 14y + 4z = k^2 - 109$

$\Rightarrow 2(x^2 + y^2 + z^2 - 2x - 7y + 2z) = k^2 - 109$

$\Rightarrow x^2 + y^2 + z^2 - 2x - 7y + 2z = \frac{k^2-109}{2}$

29. Here, $a = 3x^2$, $b = = \frac{-1}{3x}$ and n = 9

We have a formula,

$t_{r+1} = \binom{10}{r} a^{n-r} b^r$

$= \binom{9}{r} (3x^2)^{9-r} \left(\frac{-1}{3x}\right)^r$

$= \binom{9}{r} (3)^{9-r} (x^2)^{9-r} \left(\frac{-1}{3}\right)^r (x)^{-r}$

$= \binom{9}{r} (3x)^{9-r} (x)^{18-2r} \left(\frac{-1}{3}\right)^r (x)^{-r}$

$= \binom{9}{r} (3x)^{9-r} (x)^{18-2r-r} \left(\frac{-1}{3}\right)$

$= \binom{9}{r} (3x)^{9-r} \left(\frac{-1}{3}\right)^r (x)^{18-3r}$

To get coefficient of x^6 we must have,

$(x)^{18-3r} = x^6$

18 - 3r = 6

3r = 12

r = 4

Therefore, coefficient of $x^6 = \binom{9}{4} (3)^{9-4} (3)^{9-4} \left(\frac{-1}{3}\right)^4$

$= \frac{9\times8\times7\times6}{4\times3\times2\times1} \cdot (3)^5 \left(\frac{1}{3}\right)^4$

$= 126 \times 3$

= 378

Hence the coefficient of x^6 = 378..

OR

To find: Expansion of $\left(x^2 - \frac{3x}{7}\right)^7$

Formula used: ${}^nC_r = \frac{n!}{(n-r)!(r)!}$

We know that ,$(a + b)^n = {}^nC_0a^n + {}^nC_1a^{n-1}b + {}^nC_2a^{n-2}b^2 + \ldots\ldots + {}^nC_{n-1}ab^{n-1} + {}^nC_nb^n$

Here We have, $(x^2 - \frac{3x}{7})^7$

$\Rightarrow \left[{}^7C_0\left(x^2\right)^{7-0}\right] + \left[7C_1\left(x^2\right)^{7-1}\left(-\frac{3x}{7}\right)^1\right] + \left[7c_2\left(x^2\right)^{7-2}\left(-\frac{3x}{7}\right)^2\right] + \left[7C_3\left(x^2\right)^{7-3}\left(-\frac{3x}{7}\right)^3\right] + \left[7C_4\left(x^2\right)^{7-4}\left(-\frac{3x}{7}\right)^4\right]$

$+ \left[7C_5\left(x^2\right)^{7-5}\left(-\frac{3x}{7}\right)^5\right] + \left[7C_6\left(x^2\right)^{7-6}\left(-\frac{3x}{7}\right)^6\right] + \left[7C_7\left(-\frac{3x}{7}\right)^7\right]$

$\Rightarrow \left[\frac{7!}{0!(7-0)!}\left(x^2\right)^7\right] - \left[\frac{7!}{1!(7-1)!}\left(x^2\right)^6\left(\frac{3x}{7}\right)\right] + \left[\frac{7!}{2!(7-2)!}\left(x^2\right)^5\left(\frac{9x^2}{49}\right)\right] - \left[\frac{7!}{3!(7-3)!}\left(x^2\right)^4\left(\frac{27x^3}{343}\right)\right]$

$+ \left[\frac{7!}{4!(7-4)!}\left(x^2\right)^3\left(\frac{81x^4}{2401}\right)\right] - \left[\frac{7!}{5!(7-5)!}\left(x^2\right)^2\left(\frac{243x^5}{16807}\right)\right] + \left[\frac{7!}{6!(7-6)!}\left(x^2\right)^1\left(\frac{729x^6}{117649}\right)\right] - \left[\frac{7!}{7!(7-7)!}\left(\frac{2187x^7}{823543}\right)\right]$

$- \left[\frac{7!}{7!(7-7)!}\left(\frac{2187x^7}{823543}\right)\right] + \left[21\left(x^{10}\right)\left(\frac{9x^2}{49}\right)\right] - \left[35\left(x^8\right)\left(\frac{27x^3}{343}\right)\right]$

$+ \left[35\left(x^6\right)\left(\frac{81x^4}{2401}\right)\right] - \left[21\left(x^4\right)\left(\frac{243x^5}{16807}\right)\right] + \left[7\left(x^2\right)\left(\frac{729x^6}{117649}\right)\right] - \left[1\left(\frac{2187x^7}{823543}\right)\right]$

$\Rightarrow x^{24} - 3x^{13} + \left(\frac{27}{7}\right)x^{12} - \left(\frac{135}{49}\right)x^{11} + \left(\frac{405}{343}\right)x^{10} - \left(\frac{729}{2401}\right)x^9 + \left(\frac{729}{16807}\right)x^8 - \left(\frac{2187}{823543}\right)x^7$

$x^{14} - 3x^{13} + \left(\frac{27}{7}\right)x^{12} - \left(\frac{135}{49}\right)x^{11} + \left(\frac{405}{343}\right)x^{10} - \left(\frac{729}{2401}\right)x^9 + \left(\frac{729}{16807}\right)x^8 - \left(\frac{2187}{823543}\right)x^7$

30. M.I. of $= \sqrt{5} + 3i$

$= \frac{1}{\sqrt{5}+3i} = \frac{1}{\sqrt{5}+3i} \times \frac{\sqrt{5}-3i}{\sqrt{5}-3i}$

$= \frac{\sqrt{5}-3i}{(\sqrt{5})^2-(3i)^2}$

$= \frac{\sqrt{5}-3i}{5-9i^2} = \frac{\sqrt{5}-3i}{5+9} = \frac{1}{14}(\sqrt{5} - 3i)$

OR

Here $\overline{-6 - 24i} = -6 + 24i$

Now (x - iy) (3 + 5i) = -6 + 24i

$\Rightarrow 3x + 5xi - 3yi - 5yi^2 = 6 + 24i$

$\Rightarrow (3x + 5y) + (5x - 3y)i = -6 + 24i$

Comparing both sides, we have

3x + 5y = -6. . . . (i)

and 5x - 3y = 24 .... (ii)

Multiplying (i) by 3 and (ii) by 5 and then adding

$9x + 15y = -18$

$\underline{25x - 15y = 120} \Rightarrow x = 3$

$34x = 102$

Putting x = 3 in (i)

3(3)+5y=-6

Thus y=-3

31. Let $A \cup B = A \cap B$

Then, we have to prove that A = B

Let $x \in A \Rightarrow x \in A \cup B$

$\Rightarrow x \in A \cap B$ $(\because A \cup B = A \cap B)$

$\Rightarrow x \in A \, and \, x \in B$

$\Rightarrow x \in B$

$\therefore A \subseteq B$ ...... (1)

Let $x \in B \Rightarrow x \in A \Rightarrow x \in A \cup B$

$\Rightarrow x \in A \cap B$ $(\because A \cup B = A \cap B)$

$\Rightarrow x \in A$ and $x \in B$

$\Rightarrow$ x $\in$ A

$\therefore B \subseteq A$....... (ii)

From (i) and (ii), we get

A = B

Thus $A \cup B = A \cap B \Rightarrow A = B$ ..... (iii)

Now let A = B

Thus, we have to prove that $A \cup B = A \cap B$

$\therefore A \cup B = A$ and $A \cap B = A$

$\Rightarrow A \cup B = A \cap B$

Thus A = B$\Rightarrow A \cup B = A \cap B$ ..... (iv)

From (iii) and (iv), we get

$A \cup B = A \cap B \Leftrightarrow A = B$

Section D

32. Bag contains:

6 -Red balls

4 -White balls

8 -Blue balls

Since three ball are drawn,

$\therefore n(S) = {}^{18}C_3$

i. Let E be the event that one red and two white balls are drawn.

$\therefore n(E) = {}^{6}C_1 \times {}^{4}C_2$

$\therefore P(E) = \frac{{}^6C_1 \times {}^4C_2}{{}^{18}C_3} = \frac{6\times4\times3}{2} \times \frac{3\times2}{18\times17\times16}$

$P(E) = \frac{3}{68}$

ii. Let E be the event that two blue balls and one red ball was drawn.

$\therefore n(E) = {}^{8}C_2 \times {}^{6}C_1$

$\therefore P(E) = \frac{{}^8C_2 \times {}^6C_1}{{}^{18}C_3} = \frac{8\times7}{2} \times 6 \times \frac{3\times2\times1}{18\times17\times16} = \frac{7}{34}$

$P(E) = \frac{7}{34}$

iii. Let E be the event that one of the ball must be red.

$\therefore$ E = {(R,W,B) or (R,W,W) or (R,B,B)}

$\therefore n(E) = {}^{6}C_1 \times {}^{4}C_1 \times {}^{8}C_1 + {}^{6}C_1 \times {}^{4}C_2 + {}^{6}C_1 \times {}^{8}C_2$

$\therefore P(E) = \frac{{}^6C_1 \times {}^4C_1 \times {}^8C_1 + {}^6C_1 \times {}^4C_2 + {}^6C_1 \times {}^8C_2}{{}^{18}C_3} = \frac{6\times4\times8 + \frac{6\times4\times3}{2\times1} + \frac{6\times8\times7}{2\times1}}{\frac{18\times17\times16}{3\times2\times1}}$

$= \frac{396}{816} = \frac{33}{68}$

33. Clearly,

cos x cos 2x cos 3x = $\frac{1}{2}$ {2 cos x cos 2x cos 3x}

= $\frac{1}{2}$ {(2 cos x cos 2x) cos 3x}

= $\frac{1}{2}$ {(cos 3x + cosx) cos 3x}

= $\frac{1}{2}$ {$\cos^2$ 3x + cos 3x cos x}

= $\frac{1}{4}$ {2 $\cos^2$ 3x + 2 cos 3x cos x}

= $\frac{1}{4}$ {1 + cos 6x + cos 4x + cos 2x}

$\therefore \lim_{x\to0} \frac{1-\cos x \cos 2x \cos 3x}{\sin^2 2x}$

$= \lim_{x\to0} \frac{1-\frac{1}{4}(1+\cos 6x+\cos 4x+\cos 2x)}{\sin^2 2x}$

$= \lim_{x\to0} \frac{4-1-\cos 6x-\cos 4x-\cos 2x}{4\sin^2 2x}$

$= \lim_{x\to0} \frac{(1-\cos 6x)+(1-\cos 4x)+(1-\cos 2x)}{4\sin^2 2x}$

$= \lim_{x\to0} \frac{2\sin^2 3x+2\sin^2 2x+2\sin^2 x}{4\sin^2 2x}$

$= \lim_{x\to0} \frac{\frac{\sin^2 3x}{x^2}+\frac{\sin^2 2x}{x^2}+\frac{\sin^2 x}{x^2}}{2\left(\frac{\sin^2 2x}{x^2}\right)}$

$= \lim_{x\to0} \frac{\left(\frac{\sin 3x}{x}\right)^2+\left(\frac{\sin 2x}{x}\right)^2+\left(\frac{\sin x}{x}\right)^2}{2\left(\frac{\sin 2x}{x}\right)^2}$

$= \lim_{x\to0} \frac{9\times\left(\frac{\sin 3x}{3x}\right)^2+4\times\left(\frac{\sin 2x}{2x}\right)^2+\left(\frac{\sin x}{x}\right)^2}{2\times4\left(\frac{\sin 2x}{2x}\right)^2}$

$= \frac{9\times1+4\times1+1}{8} = \frac{14}{8} = \frac{7}{4}$

OR

We have to find the derivative of f(x) = $\frac{\cos x}{x}$

Derivative of a function f(x) is given by f'(x) = $\lim\limits_{h\to 0}$ = $\frac{f(x+h)-f(x)}{h}$ {where h is a very small positive number}

∴ Derivative of f(x) = $\frac{\cos x}{x}$ is given as f'(x) = $\lim\limits_{h\to 0}$ = $\frac{f(x+h)-f(x)}{h}$

$\Rightarrow \mathrm{f(x)} = \lim\limits_{\mathrm{h}\to 0} \frac{\frac{\cos(x+h)}{x+h} - \frac{\cos x}{x}}{h}$

$\Rightarrow \mathrm{f(x)} = \lim\limits_{\mathrm{h}\to 0} \frac{\frac{x\cos(x+h)-(x+h)\cos x}{x(x+h)}}{h} = \lim\limits_{h\to 0} \frac{x\cos(x+h)-(x+h)\cos x}{h(x)(x+h)}$

Using the algebra of limits we have:

$\Rightarrow \mathrm{f(x)} = \lim\limits_{\mathrm{h}\to 0} \frac{x\cos(x+h)-(x+h)\cos x}{h} \times \lim\limits_{h\to 0} \frac{1}{x(x+h)}$

$\Rightarrow \mathrm{f(x)} = \lim\limits_{\mathrm{h}\to 0} \frac{x\cos(x+h)-(x+h)\cos x}{h} \times \frac{1}{x(x+o)}$

$\Rightarrow \mathrm{f(x)} = \frac{1}{\mathrm{x}^2} \lim\limits_{\mathrm{h}\to 0} \frac{x\cos(x+h)-(x+h)\cos x}{h}$

$\Rightarrow \mathrm{f(x)} = \frac{1}{\mathrm{x}^2} \lim\limits_{\mathrm{h}\to 0} \frac{x\cos(x+h)-x\cos x-h\cos x}{h}$

Using the algebra of limits, we have:

$\Rightarrow \mathrm{f(x)} = \frac{1}{\mathrm{x}^2} \left\{ \lim\limits_{h\to 0} \frac{-h\cos x}{h} + \lim\limits_{h\to 0} \frac{x\cos(x+h)-x\cos x}{h} \right\}$

$\Rightarrow \mathrm{f(x)} = \frac{1}{\mathrm{x}^2} \left\{ -\lim\limits_{h\to 0} \cos x + \lim\limits_{h\to 0} \frac{x(\cos(x+h)-\cos x)}{h} \right\}$

Using the algebra of limits we have:

∴ f'(x) = $-\frac{\cos x}{x^2} + \frac{1}{x} \lim\limits_{h\to 0} \frac{\cos(x+h)-\cos x}{h}$

We can't evaluate the limits at this stage only as on putting value it will take $\frac{0}{0}$ form. So, we need to do little modifications.

Use: cos A – cos B = – 2 sin ($\frac{(A+B)}{2}$) sin ($\frac{(A-B)}{2}$)

∴ f'(x) = $-\frac{\cos x}{x^2} + \frac{1}{x} \lim\limits_{h\to 0} \frac{-2\sin\left(\frac{2x+h}{2}\right)\sin\left(\frac{h}{2}\right)}{h}$

$\Rightarrow$ f'(x) = $-\frac{\cos x}{x^2} + \frac{1}{x} \lim\limits_{h\to 0} \frac{\sin\left(x+\frac{h}{2}\right)\sin\left(\frac{h}{2}\right)}{\frac{h}{2}}$

Using algebra of limits:

$\Rightarrow$ f'(x) = $-\frac{\cos x}{x^2} + \frac{1}{x} \lim\limits_{h\to 0} \frac{\sin\left(\frac{h}{2}\right)}{\frac{h}{2}} \times \lim\limits_{h\to 0} \sin\left(x+\frac{h}{2}\right)$

By using the formula we get : $\lim\limits_{x\to 0} \frac{\sin x}{x} = 1$

$\Rightarrow$f'(x) =$-\frac{\cos x}{x^2} + \frac{1}{x} \lim\limits_{h\to 0} \sin\left(x+\frac{h}{2}\right)$

Put the value of h to evaluate the limit:

∴ f'(x) =$-\frac{\cos x}{x^2} + \frac{1}{x} \times$ sin (x + 0) = $-\frac{\cos x}{x^2} - \frac{\sin x}{x}$

Hence,

Derivative of f(x) = (cos x)/x is $-\frac{\cos x}{x^2} - \frac{\sin x}{x}$

34. Let the four numbers in GP be

$\frac{a}{r^3}, \frac{a}{r}$, ar, ar^3 ...(i)

Product of four numbers = 4096 [given]

$\Rightarrow \left(\frac{a}{r^3}\right)\left(\frac{a}{r}\right)$ (ar) (ar^3) = 4096

$\Rightarrow a^4 = 4096 \Rightarrow a^4 = 8^4$

On comparing the base of the power 4, we get

$\Rightarrow \frac{a}{r^3} + \frac{a}{r}$ + ar + ar^3 = 85

$\Rightarrow$ a $\left[\frac{1}{r^3} + \frac{1}{r} + r + r^3\right]$ = 85

$\Rightarrow$ 8 [$r^3 + \frac{1}{r^3}$] + 8 [r + $\frac{1}{r}$] = 85 [∵ a = 8]

$\Rightarrow 8\left[\left(r+\frac{1}{r}\right)^3 - 3\left(r+\frac{1}{r}\right)\right] + 8\left(r+\frac{1}{r}\right)$ = 85 [∵ $a^3 + b^3 = (a + b)^3 - 3(a + b)$]

$\Rightarrow 8\left(r+\frac{1}{r}\right)^3 - 16\left(r+\frac{1}{r}\right) - 85 = 0$ ...(ii)

On putiing $\left(r+\frac{1}{r}\right)$ = x in Eq. (ii), we get

$8x^3 - 16x - 85 = 0$

$\Rightarrow (2x - 5)(4x^2 + 10x + 17) = 0$

$\Rightarrow 2x - 5 = 0$ [$\because 4x^2 + 10x + 177 = 0$ has imaginary roots]

$\Rightarrow x = \frac{5}{2} \Rightarrow r + \frac{1}{r} = \frac{5}{2}$ [put $x = r + \frac{1}{r}$]

$\Rightarrow 2r^2 - 5r + 2 = 0$

$\Rightarrow (r - 2)(2r - 1) = 0$

$\Rightarrow r = 2$ or $r = \frac{1}{2}$

On putting $a = 8$ and $r = 2$ or $r = \frac{1}{2}$ in Eq. (i), we obtain four numbers as

$\frac{8}{2^3}, \frac{8}{2}, 8 \times 2, 8 \times 2^3$

or $\frac{8}{(1/2)^3}, \frac{8}{(1/2)}, 8 \times \frac{1}{2}, 8 \times \left(\frac{1}{2}\right)^3$

$\Rightarrow$ 1, 4, 16, 64 or 64, 16, 4, 1.

35. We have to prove that $\sin 5x = 5 \sin x - 20 \sin^3 x + 16 \sin^5 x$.

Let us consider LHS = $\sin 5x$

$\sin 5x = \sin(3x + 2x)$

But we know,

$\sin(x + y) = \sin x \cos y + \cos x \sin y$ ... (i)

$\Rightarrow \sin 5x = \sin 3x \cos 2x + \cos 3x \sin 2x$

$\Rightarrow \sin 5x = \sin(2x + x)\cos 2x + \cos(2x + x)\sin 2x$ ... (ii)

And

$\cos(x + y) = \cos(x)\cos(y) - \sin(x)\sin(y)$ ... (iii)

Now substituting equation (i) and (iii) in equation (ii), we get

$\Rightarrow \sin 5x = (\sin 2x \cos x + \cos 2x \sin x)\cos 2x + (\cos 2x \cos x - \sin 2x \sin x)\sin 2x$

$\Rightarrow \sin 5x = \sin 2x \cos 2x \cos x + \cos^2 2x \sin x + (\sin 2x \cos 2x \cos x - \sin^2 2x \sin x)$

$\Rightarrow \sin 5x = 2\sin 2x \cos 2x \cos x + \cos^2 2x \sin x - \sin^2 2x \sin x$ ... (iv)

Now $\sin 2x = 2\sin x \cos x$ ... (v)

And $\cos 2x = \cos^2 x - \sin^2 x$ ... (vi)

Substituting equation (v) and (vi) in equation (iv), we get

$\Rightarrow \sin 5x = 2(2\sin x \cos x)(\cos^2 x - \sin^2 x)\cos x + (\cos^2 x - \sin^2 x)^2 \sin x - (2\sin x \cos x)^2 \sin x$

$\Rightarrow \sin 5x = 4(\sin x \cos^2 x)([1 - \sin^2 x] - \sin^2 x) + ([1 - \sin^2 x] - \sin^2 x)^2 \sin x - (4\sin^2 x \cos^2 x)\sin x$ (as $\cos^2 x + \sin^2 x = 1 \Rightarrow \cos^2 x = 1 - \sin^2 x$)

$\Rightarrow \sin 5x = 4(\sin x [1 - \sin^2 x])(1 - 2\sin^2 x) + (1 - 2\sin^2 x)^2 \sin x - 4\sin^3 x [1 - \sin^2 x]$

$\Rightarrow \sin 5x = 4\sin x(1 - \sin^2 x)(1 - 2\sin^2 x) + (1 - 4\sin^2 x + 4\sin^4 x)\sin x - 4\sin^3 x + 4\sin^5 x$

$\Rightarrow \sin 5x = (4\sin x - 4\sin^3 x)(1 - 2\sin^2 x) + \sin x - 4\sin^3 x + 4\sin^5 x - 4\sin^3 x + 4\sin^5 x$

$\Rightarrow \sin 5x = 4\sin x - 8\sin^3 x - 4\sin^3 x + 8\sin^5 x + \sin x - 8\sin^3 x + 8\sin^5 x$

$\Rightarrow \sin 5x = 5\sin x - 20\sin^3 x + 16\sin^5 x$

Hence LHS = RHS

Hence proved.

OR

We have,

LHS = $\cos 20° \cos 40° \cos 60° \cos 80°$

$\Rightarrow$ LHS = $\cos 60° (\cos 20° \cos 40°) \cos 80°$

$\Rightarrow$ LHS = $\frac{1}{2} \times \frac{1}{2}(2 \cos 20° \cos 40°) \cos 80°$ $\left[\because \cos\frac{\pi}{3} = \frac{1}{2}\right]$

$\Rightarrow$ LHS = $\frac{1}{4}$ [{$\cos(40° + 20°) + \cos(40° - 20°)$} $\cos 80°$] [$\because 2\cos A \cos B = \cos(A + B) + \cos(A - B)$]

$\Rightarrow$ LHS = $\frac{1}{4}$ {$(\cos 60° + \cos 20°)\cos 80°$}

$\Rightarrow$ LHS = $\frac{1}{4}$ {$(\frac{1}{2} + \cos 20°)\cos 80°$}

$\Rightarrow$ LHS = $\frac{1}{4}$ {$\frac{1}{2}\cos 80° + \cos 80° \cos 20°$}

$\Rightarrow$ LHS = $\frac{1}{8}$ {$\cos 80° + 2\cos 80° \cos 20°$}

$\Rightarrow$ LHS = $\frac{1}{8}$ [$\cos 80°$ + {$\cos(80° + 20°) + \cos(80° - 20°)$}]

$\Rightarrow$ LHS = $\frac{1}{8}$ {cos 80° + cos 100° + cos 60°}
$\Rightarrow$ LHS = $\frac{1}{8}$ {cos 80° + cos (180° - 80°) + cos 60°}
$\Rightarrow$ LHS = $\frac{1}{8}$ {cos 80° - cos 80° + cos 60°} [$\because$ cos (180° - x) = -cos x]
$\Rightarrow$ LHS $= \frac{1}{8} \times \frac{1}{2} = \frac{1}{16}$ = RHS

Section E

36. i. From the diagram equation of parabola is $x^2 = -4ay$
Vertex is 10m hight and spam is 100m
parabola passes through (50, -10)
Hence, $50^2 = -4a(-10)$
$\Rightarrow$ 2500 = 40a
$\Rightarrow$ a = $\frac{2500}{40} = 62.5$
Hence coordinates of focus = (-a, 0) = (-62.5, 0)
ii. Equation of parabola is $x^2 = -4ay$ and $a = \frac{2500}{40} = 62.5$
Equation is $x^2 = -4\left(\frac{2500}{40}\right)y$
$\Rightarrow$ $x^2 = -250y$
Length of latus rectum is 4a = 4 $\times$ 62.5 = 250m
iii. Equation parabola $x^2 = -250y$
Coordinates of the point at 20 m from mid point = (20, y)
Substituting in the equation of parabola
$\Rightarrow$ 400 = -250y
$\Rightarrow$ y = $\frac{-400}{250}$ = -1.6
height of the bridge = 10 - 1.6 = 8.4m
OR
vertex of parabola is (0, 0) and focus is (0, -62.5)
$\Rightarrow$ (0, -62.5) is center and (0, 0) is on the circle
$\Rightarrow$ r = 0 - (-62.5) = 62.5 m

37. i. coefficient of variation of wages, of firm A = 0.19
coefficient of variation of wages, of firm B = $\frac{121}{5253} \times 100$ = 0.21
$\therefore$ Firm B shows greater variability in individual wages.
ii. Standard deviation, $\sigma = \sqrt{\sigma^2} = \sqrt{121}$ = 11
iii. Variance of distribution of wages, σ^2 = 100
Standard deviation, $\sigma = \sqrt{\sigma^2} = \sqrt{100}$ = 10
coefficient of Variation = $\frac{\sigma}{\bar{x}} \times 100$
= $\frac{10}{5,253} \times 100$
= 0.19
OR
No. of wage earners = 586
Mean of monthly wages, $\bar{x}$ = ₹5253
Amount paid by firm A = ₹(586 $\times$ 5253) = ₹3078258

38. i. x- intercept = a = 20, y- intercept = b = 15
By intercept form of equation of line
$\frac{x}{a} + \frac{y}{b} = 1$
$\Rightarrow \frac{x}{20} + \frac{y}{15} = 1$
$\Rightarrow$ 15x + 20y = 300
$\Rightarrow$ 3x + 4y = 60
ii. x - 2y = 20
x-intercept = a = 20, y-intercept = b = -10
By intercept form of erquation of line $\frac{x}{a} + \frac{y}{b} = 1$
$\Rightarrow \frac{x}{20} - \frac{y}{10} = 1$

$\Rightarrow$ 10x - 20y = 200

$\Rightarrow$ x - 2y = 20

iii. from given BC = OB + OC = 15 + 10 = 25 km

OA = 20 km

Area $\triangle$ABC = $\frac{1}{2} \times BC \times OA$

$\Rightarrow$ Area $\triangle$ABC = $\frac{1}{2} \times 25 \times 20$

$\Rightarrow$ Area $\triangle$ABC = 250 km^2

OR

A = (20, 0) and C = (0, -10)

by distance formula

AC = $\sqrt{(20-0)^2+(0+10)^2}$

$= \sqrt{400+100}$

$= \sqrt{500} = 10\sqrt{5}$ km